15/2/18 Tema 4, Es 4

Sono dati $P=(2,0,1)$, $Q=(1,1,1)$ e

$$r = \begin{cases} x-y+z-3=0 \\ 2x+3y+z-2=0 \end{cases}$$

a) Trovare eq cartesiana per π_1 contenente r e P

b) " " $\pi_2 \supset r$, $\pi_2 \parallel PQ$

<u>Soluzione</u>

$$\lambda(x-y+z-3)+\mu(2x+3y+z-2)=0 \quad \circledast$$

$$x(\lambda+2\mu)+y(3\mu-\lambda)+z(\lambda+\mu)-3\lambda-2\mu=0$$

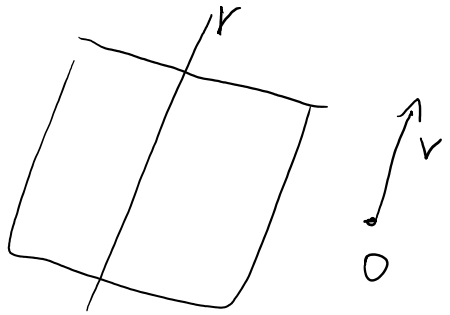
$$2(\lambda+2\mu)+\lambda+\mu-3\lambda-2\mu=0 \iff 3\mu=0 \iff \mu=0 \rightsquigarrow \lambda=1$$

$\pi_1: x-y+z-3=0$

b) $PQ = P+\langle v\rangle$ $\qquad v=P-Q=(1,-1,0)$

$r \to r = R+\langle w\rangle \implies \pi_2 : R+\langle w, v\rangle$

Imponendo il parallelismo di v con $\circledast$



$$\lambda(x-y+z)+\mu(2x+3y+z)=0$$

$$\lambda(1-(-1))+\mu(2-3)=0$$

$$2\lambda=\mu$$

$\mu = 2,\ \lambda = 1$ li sostituiamo in ⊛

$$\pi_2:\ x - y + z - 3 + 4x + 6y + 2z - 4 = 0$$

$$5x + 5y + 3z - 7 = 0$$

31/1/'18 Tema 4, Es4

Sono dati $P = (1, 0, 1)$, $Q = (-1, -1, -1)$, $R = (-1, 1, 2)$, $S = (3, 1, 3)$

a) Trovare la distanza di P dalla retta RS

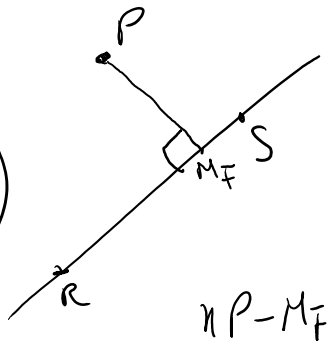
b) " " tra PQ e RS

<u>Soluzione</u>

$RS = R + \langle S - R \rangle \qquad S - R = (4, 0, 1)$

$\Downarrow$

$M_t = (-1 + 4t, 1, 2 + t) \left(\xrightarrow{t = -k} (-1 - 4k, 1, 2 - k) \right)$



$\|P - M_{\bar{t}}\|$

$PM_t = (-2 + 4t, 1, 1 + t) \perp S - R$

$PM_t \cdot (S - R) =$

$= -8 + 16t + 1 + t = 0 \iff 17t = 7 \iff \bar{t} = 7/17$

$$PM_{7/17} = \left(-2 + \frac{28}{17} = -\frac{6}{17},\ 1,\ \frac{24}{17} \right)$$

$$\|PM_{7/17}\| = \left\| \left(-\frac{6}{17}, \frac{17}{17}, \frac{24}{17} \right) \right\| = \frac{1}{17} \sqrt{36 + 289 + 576} =$$

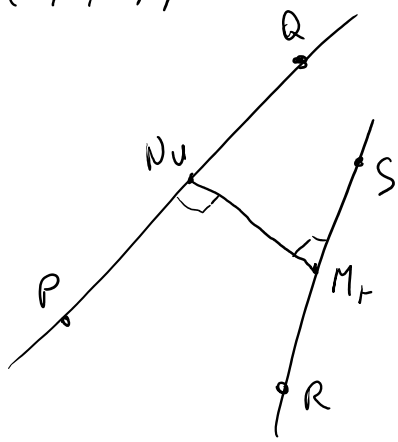
$$= \boxed{\frac{1}{17} \sqrt{901}} = \frac{1}{17} \sqrt{17 \cdot 53} = \sqrt{\frac{53}{17}}$$

$= \sqrt{17}$

b) $PQ = P + \langle P-Q \rangle = (1,0,1) + \langle (2,1,2) \rangle$

$\Downarrow$

$N_u = (1+2u,\ u,\ 1+2u)$



$N_u M_t = (4t+2u-2,\ u+1,\ t+2u+1)$

$N_u M_t \perp (P-Q)$

$N_u M_t \perp (R-S)$

$\begin{cases} N_u M_t \cdot (P-Q) = 0 \\ N_u M_t \cdot (R-S) = 0 \end{cases}$ $\quad \begin{cases} 8t+4u-4+u+1+2t+4u+2=0 \\ 16t+8u-8+t+2u+1=0 \end{cases}$

$\begin{cases} 10t+9u=1 \\ 17t+10u=7 \end{cases}$ $\quad \text{II}-\text{I} \begin{cases} 7t+u=6 \iff u=6-7t \\ 10t+9(6-7t)=1 \end{cases}$

$54-53t=1 \iff 53=53t \iff t=1 \implies u=-1$

$N_{-1} M_1 = (4-2-2,\ -1+1,\ 1-2+1) = (0,0,0)$

$\implies d(PQ, RS) = 0 \implies PQ \cap RS \neq \emptyset$

$\pi: P + \langle v_1, v_2 \rangle \qquad Q = (Q_1, Q_2, Q_3)$

$P = \begin{pmatrix} p_1 \\ p_2 \\ p_3 \end{pmatrix} \quad \left(\begin{pmatrix} a \\ b \\ c \end{pmatrix}, \begin{pmatrix} d \\ e \\ f \end{pmatrix} \right)$

$P_{\mu,\lambda} = (p_1 + \mu a + \lambda d,\ p_2 + \mu b + \lambda e,\ p_3 + \mu c + \lambda f)$

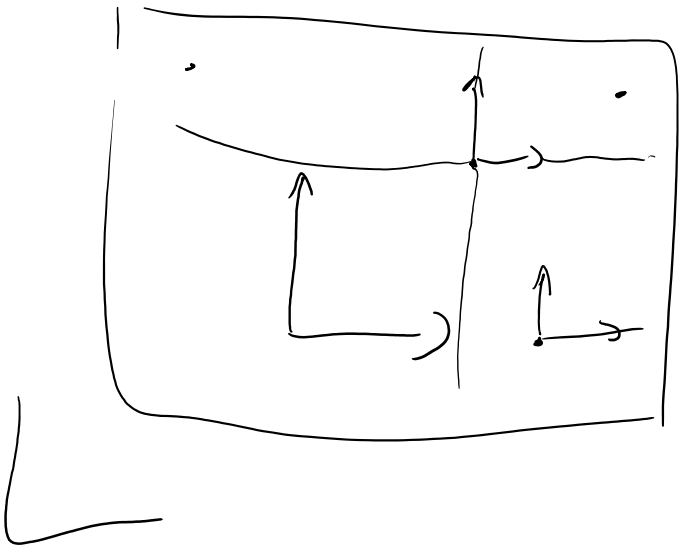
$P_{\mu,\lambda} Q = (p_1 - Q_1, \ldots)$

$P_{N,\lambda}\, Q \perp v_1, \quad P_{M,\lambda}\, Q \perp v_2$

$M: ax + \cancel{by} + \cancel{cz} = d \qquad P = \left(\frac{d}{a}, 0, 0\right)$

$ax + by + cz = 0 \to -cz = ax + by$

$z = -\frac{a}{c}x - \frac{b}{c}y \longrightarrow \begin{pmatrix} 1 \\ 0 \\ -\frac{a}{c} \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ -\frac{b}{c} \end{pmatrix}$



18/9/'17, Tema 3, Es 4

Sono dati $P=(1,1,0)$, $Q=(-1,0,-2)$, $R=(2,1,-2)$, $O=(0,0,0)$. Trovare se esistono

a) L'eq cartesiana di un piano $\pi_1 \supset OP$, $\pi_1 \parallel QR$

b) " " $\pi_2 \supset OP$, $\pi_2 \perp QR$

c) La distanza tra OP e QR

Soluzione

a) $OP = O + \langle (1,1,0) \rangle \to M_t = (t, t, 0) \to \begin{cases} x - y = 0 \\ z = 0 \end{cases}$

$\parallel$

$\langle v_1, v_2 \rangle = 0 \qquad QR = \langle (3, 1, 0) \rangle$

$\pi_{\lambda,\mu}$ $\lambda(x-y)+\mu(z)=0$ $\qquad$ $QR = \langle (3,1,0) \rangle + Q$

$2\lambda = 0 \iff \lambda = 0, \ \mu = 1 \Rightarrow \pi_1 : z = 0$

b) $\text{rk} \begin{pmatrix} \lambda & -\lambda & \mu \\ 3 & 1 & 0 \end{pmatrix} = 1$

$\langle v \rangle^\perp = \langle (0,0,1), (1,-3,0) \rangle \rightsquigarrow 3x+y=0$

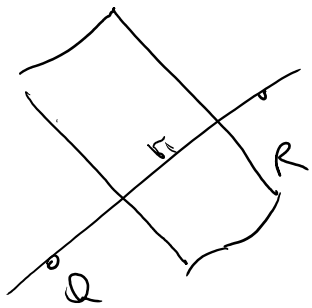
$\left[\begin{array}{l} v = (v_1, v_2, v_3) \\ \langle v \rangle^\perp : v_1 x + v_2 y + v_3 z = 0 \end{array} \right]$

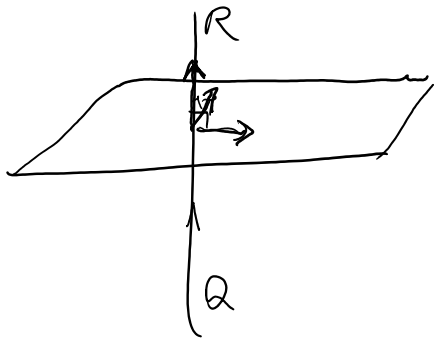
$\pi_2 = O + \langle v \rangle^\perp$

$(1,1,0) \in \langle v \rangle^\perp$

$\hookrightarrow 3+1 = 4 \neq 0 \quad P \notin \pi_2 \Rightarrow OP \notin \pi_2$

$\Rightarrow$ Un tale π_2 non può esistere





c) $d(QR, OP) = d(Q, \pi_1) = \left| \frac{-2}{1} \right| = 2$

$z = 0 \qquad \underline{|-2 \cdot 1|}$

$\sqrt{1^2}$

17/9/'18, Tema 4, Es 3

Si consideri $(\mathbb{R}^3, \cdot)$, $U = \{(x,y,z) \in \mathbb{R}^3 \mid 2x + y - z = 0\}$

a) Determinare basi ortonormali di U e di $U^\perp$

b) Sia $v = (2,0,1)$. Determinare il vettore di norma minima di $v + U$

<u>Soluzione</u>

a) $U^\perp = \langle (2,1,-1) \rangle \longrightarrow \frac{1}{\sqrt{6}}(2,1,-1)$

$z = 2x + y \rightsquigarrow U = \left\langle \overset{v_1}{\begin{pmatrix}1\\0\\2\end{pmatrix}}, \overset{v_2}{\begin{pmatrix}0\\1\\1\end{pmatrix}} \right\rangle$

$u_1 = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\1\\1\end{pmatrix}$ $\quad \tilde{u}_2 = v_1 - \frac{\overset{2}{\overset{\shortparallel}{(v_1 \cdot v_2)}}}{\underset{2}{\underset{\shortparallel}{(v_2 \cdot v_2)}}} v_2 = \begin{pmatrix}1\\0\\2\end{pmatrix} - \begin{pmatrix}0\\1\\1\end{pmatrix} = \begin{pmatrix}1\\-1\\1\end{pmatrix}$

$u_2 = \frac{1}{\sqrt{3}}\begin{pmatrix}1\\-1\\1\end{pmatrix}$

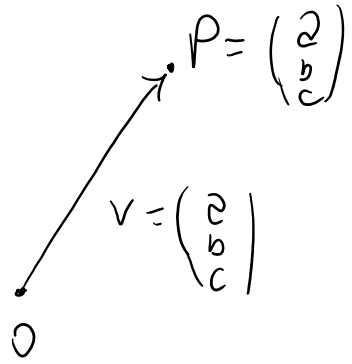
b) $\tilde{U} = \begin{pmatrix}2\\0\\1\end{pmatrix} + \left\langle \begin{pmatrix}1\\0\\2\end{pmatrix}, \begin{pmatrix}0\\1\\1\end{pmatrix} \right\rangle = v + U$

$P_{\lambda,\mu} = \begin{pmatrix}2+\lambda\\ \mu \\ 1+2\lambda+\mu\end{pmatrix} = P_{\lambda,\mu} O$

$P_{\lambda,\mu} \perp v_1 \quad , \quad P_{\lambda,\mu} \perp v_1$

$$\begin{cases} \mu + 1 + 2\lambda + \mu = 0 \\ 2 + \lambda - \cancel{\mu} + 1 + 2\lambda + \cancel{\mu} = 0 \end{cases} \qquad \begin{cases} 3\lambda = -3 \iff \lambda = -1 \\ 2\mu = -1 + 2 \iff \mu = 1/2 \end{cases}$$

$P_{-1,1/2} = (1, 1/2, -1/2)$

$d(P,Q) = \|P - Q\|$

$\|v\|$
$\|$
$\|P - O\|$